Mode Transitions Between Bound and Leaky Regimes in Lossy Dielectric Waveguides: A Numerical Investigation

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***Abstract*—This letter presents a novel numerical study of mode transitions between bound and leaky regimes in a lossy-substrate dielectric rib waveguide. An entire-domain basis Galerkin's method is employed to determine the wavenumbers of the transmission line modes. The obtained results show that as the loss tangent gradually increases from zero to infinity, the plots of the attenuation constants alter between regions of constant and linear variation. In the progress of this behavior the modes evolve from leaky to bound state, or the inverse, and their corresponding poles in the spectral plane cross the real axis.**

***Index Terms*— Guided waves, leaky waves, lossy dielectric waveguides.**

I. Introduction

OVER the past several years considerable attention has been devoted to the study of leaky modes supported by integrated microwave and optical circuits [1]–[6]. The physical mechanisms [1] and the mathematical conditions [2] describing the excitation of leaky waves are already well established in the literature. However, most of the numerical investigations conducted so far have dealt with ideal lossless structures [3]–[5].

The first thorough study of leakage phenomena in an anisotropic substrate microstrip line that takes dielectric losses into account has recently been reported [6]. The interesting new behavior discussed there is that, as the frequency increases, a leaky mode reverts to a guided one. A different approach has also been attempted by the authors in a previous work [7], where the modal solutions in a dielectric strip line are computed as a function of losses.

In this letter, we further examine the impact of material losses on the excitation of leaky waves. This is accomplished by considering the transition from a dielectric rib waveguide to a strip dielectric guide as the loss tangent of the substrate changes from zero to infinity. Close inspection of the behavior of the propagation constants and the migration paths of the poles in the complex spectral plane reveals how a guided-wave mode evolves into a leaky one and vice versa.

The numerical research presented here supplements the main conclusions of [6] and addresses the important aspect that leaky modes may disappear in highly lossy dielectric configurations, such as those consisting of semiconductors.

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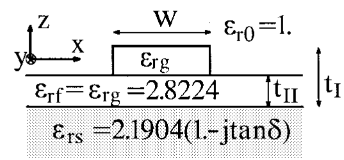

Fig. 1. Cross-sectional view of the dielectric rib waveguide considered in the analysis: $f = 30$ GHz, $\lambda_0 = 10$ mm, $t_I = 0.8\lambda_0$, $t_{II} = 0.7\lambda_0, w = 2\lambda_0, \varepsilon_{rf} = \varepsilon_{rg} = 2.8224$ and $\varepsilon_{rs} = 2.1904$ $(1 - j \tan\delta)$.

II. Theory

Fig. 1 depicts the cross section of the dielectric rib transmission line to be analyzed. The semi-infinite bottom layer is assumed to be lossy and it is characterized by the loss tangent $\tan\delta$. Consequently, a waveguide mode with $e^{j(\omega t-\beta y)}$ time and longitudinal dependence propagates along the y axis with a complex propagation wavenumber β, where $\beta = \beta_r - j\alpha, \beta_r, \alpha > 0$ and α stands for the attenuation constant.

The transmission line modes are sought by means of the electric field integral equation method [3], [4] in conjunction with Galerkin's technique. A set of entire-domain plane wave basis functions, which has been developed in [7], is then implemented. The analysis leads to the formulation of an homogeneous linear system of equations and the propagation constant is numerically computed (Muller's algorithm) by requiring the determinant of the system matrix to vanish. The elements of the matrix have the following general form [7]:

$$\int_C P(k_x, -\beta)\, dk_x. \tag{1}$$

The integration path C in the spectral k_x-plane is critically affected by the pole singularities [2] of the integrand $P(k_x, -\beta)$, which are associated with the TE and TM substrate waves [3]. Let k_s denote the wavenumber of the dominant $(\mathrm{Re}(k_s)$ is maximum) substrate mode. The corresponding pair of poles in the complex k_x-plane is given by [3]

$$k_{xs}^2 = k_s^2 - \beta^2 \Rightarrow k_{xs}^{\pm} = \pm k_{xs} \quad \text{where}$$
$$k_{xs} = \sqrt{k_s^2 - \beta^2}, \quad \mathrm{Re}(k_{xs}) > 0. \tag{2}$$

Assume now that $\mathrm{Re}(k_{xs}^2) > 0$. Depending on the material losses, $\mathrm{Im}(k_{xs}^2)$ may be positive or negative, locating the

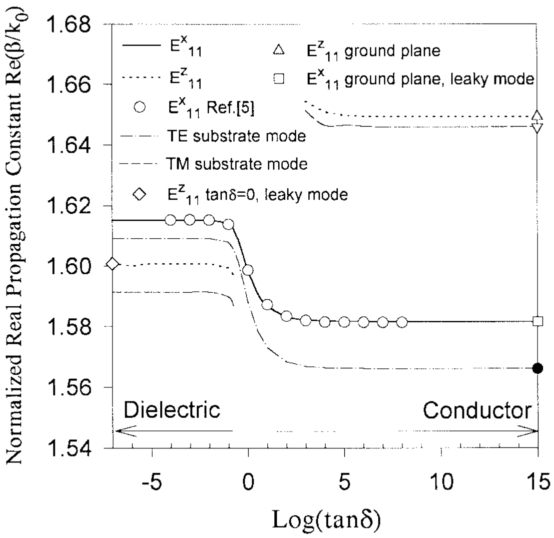

Fig. 2. Normalized real propagation constants $\mathrm{Re}(\beta)/k_0$ of the modes supported by the structure of Fig. 1, versus $\log(\tan\delta)$. The marks on the right-hand vertical axis correspond to the case $\tan\delta \to \infty$.

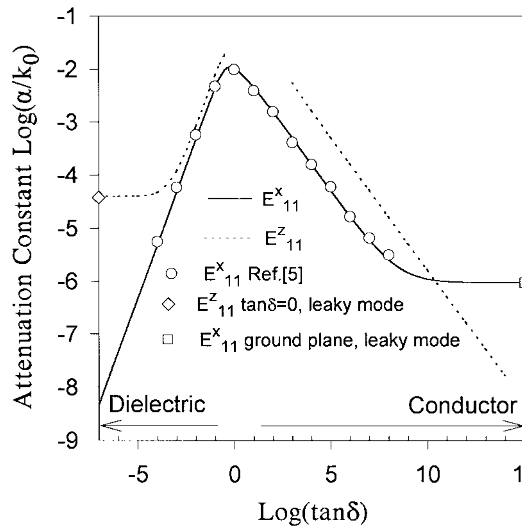

Fig. 3. Normalized attenuation constants $\log(\alpha/k_0)$ of the modes supported by the structure of Fig. 1, versus $\log(\tan\delta)$.

pole k_{xs} in the first or the fourth quadrant, respectively. The former (latter) case corresponds to exponentially increasing (decreasing) fields in the transverse x direction [3], which is the main characteristic of leaky (bound) modes. References [2] and [5] thoroughly clarify that the integration path C has to run above the pole k_{xs} and below its symmetrical $-k_{xs}$. As a concluding remark, a transmission line mode leaks into the fundamental substrate mode if the following conditions hold:

$$\mathrm{Re}(k_{xs}^2) > 0 \quad \text{and} \quad \mathrm{Im}(k_{xs}^2) > 0 \tag{3}$$

III. Results and Discussion

Referring to the transmission line of Fig. 1, the following operating conditions are considered: $f = 30$ GHz ($\lambda_0 = 2\pi/k_0 = 10$ mm), $t_I = 0.8\lambda_0, t_{II} = 0.7\lambda_0, w = 2\lambda_0, \varepsilon_{rf} = \varepsilon_{rg} = 2.8224$, and $\varepsilon_{rs} = 2.1904(1 - j\ \tan\delta)$. k_0 stands for the free-space wavenumber. We investigate the evolution of the transmission line modes as the loss tangent $\tan\delta$ varies from zero to 10^{15}, that is, when the dielectric bottom layer gradually changes to metallic state. Note that a typical value of the loss tangent for metals [8] at microwave frequencies is 10^8.

Fig. 2 plots the normalized real parts $\mathrm{Re}(\beta)/k_0$ and Fig. 3 the attenuation constants $\log(\alpha/k_0)$ of the various modes supported by the structure of Fig. 1, versus $\log(\tan\delta)$. Reference [5] presents a similar study of the E_{11}^x mode, when $\log(\tan\delta) < 8$. For comparison, these results are indicated in Figs. 2 and 3 by small circles. In this letter, we extend the study to the E_{11}^z transmission line mode and the first TE and TM substrate modes. Additionally, the value of $\tan\delta$ is increased beyond 10^8, up to 10^{15}, so that leakage phenomena will come into light.

The cases of a lossless rib waveguide ($\tan\delta = 0$ or $\log(\tan\delta) \to -\infty$) and a strip dielectric guide mounted on a ground plane ($\log(\tan\delta) \to \infty$) have also been examined and the corresponding results are marked on the left- and right-hand vertical axes in Figs. 2 and 3. Note that when $\tan\delta = 0$ ($\tan\delta \to \infty$) the E_{11}^z (E_{11}^x) mode is leaky. Between these extreme cases there exist two regions in Fig. 2, where the real parts of the propagation constants remain almost constant. In the first, $\log(\tan\delta) < -1$, the wavenumbers are practically the same as in the lossless case and the modes appear in the following order: E_{11}^x, TE, E_{11}^z, and TM. In the second, $\log(\tan\delta) > 3$, the order of the modes is: E_{11}^z, TM, E_{11}^x and TE, and the propagation constants approach their final values obtained for $\log(\tan\delta) \to \infty$.

In the intermediate region $-1 < \log(\tan\delta) < 3$ the E_{11}^x and TE modes continuously decrease between their limiting values, while the E_{11}^z and TM modes seem to drop above the E_{11}^x and TE modes. The authors lost track of the E_{11}^z and TM modes in this region. A similar problem has also been observed in [5], in a somewhat different configuration. This behavior may be attributed to the fact that as the substrate transforms to a metal, the field is pushed out of it and the TM-like modes are affected more strongly than the TE-like modes.

We next study the attenuation constants in Fig. 3. For $\tan\delta = 0$, the E_{11}^z mode leaks into the TE substrate mode and $\log(\alpha/k_0) = -4.42$. This value does not depend on the loss tangent as far as $\log(\tan\delta) < -4$. When $\tan\delta$ increases further, the attenuation constant begins to increase proportional, indicating that material losses dominate over leakage. Hence, in the region $-3 < \log(\tan\delta) < -1$ the E_{11}^z mode turns from leaky to bound state. Since a similar case has been examined in [7], we will place more emphasis on the discussion of the inverse transition, presented in the next paragraph. Finally, for $\log(\tan\delta) > 3$ the attenuation constant α/k_0 decreases proportional with $\tan\delta$ and for $\tan\delta \to \infty$ it becomes zero.

The attenuation constant of the E_{11}^x mode presents a different behavior. In the lossless case it is zero, since the mode is guided. As losses increase ($-7 < \log(\tan\delta) < -1$) it increases linearly, reaches a maximum at $\log(\tan\delta) \cong 0$ and then decreases monotonically as $\log(\tan\delta)$ increases further up to

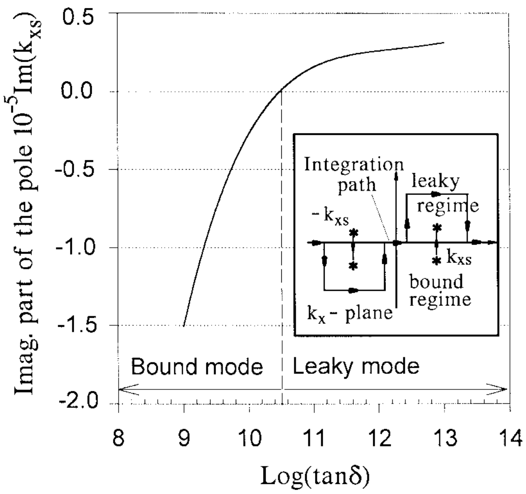

Fig. 4. Imaginary part of the pole $k_{xs} = \sqrt{k_{s,\mathrm{TM}}^2 - \beta^2}$ of the E_{11}^x mode as function of $\log(\tan\delta)$ and transition from bound to leaky regime.

8. At this point, leakage effects have not yet turned on and the researchers in [5] came to the conclusion that in the extreme case $\tan\delta \rightarrow \infty$, the attenuation constant of the E_{11}^x mode becomes zero.

However, a completely different picture appears in Fig. 3. When $\log(\tan\delta) > 8$, the slope of the curve gradually decreases to zero and for $\log(\tan\delta) > 11$ the attenuation constant is not more affected by the loss tangent. The final situation $(\tan\delta \rightarrow \infty)$ is that the E_{11}^x mode leaks energy into the TM substrate wave. In the transition region $9 < \log(\tan\delta) < 11$, the E_{11}^x mode ceases to be guided and enters into the leaky state. This can be verified in Fig. 4, where the imaginary part of the corresponding pole k_{xs} is plotted versus $\log(\tan\delta)$. It is clear that at $\log(\tan\delta) = 10.5$ the pole k_{xs} crosses the real k_x axis migrating from the fourth to the first quadrant, thus rendering the E_{11}^x mode leaky. It should be pointed out that in the region $3 < \log(\tan\delta) < 10.5$ the mode is guided even though (Fig. 2) $\mathrm{Re}(\beta) < \mathrm{Re}(k_s^{\mathrm{TM}})$.

An interesting remark that can be inferred from the above discussion is that in order to describe a metallic material in terms of dielectric parameters one has to assume sufficiently large values of the loss tangent, otherwise leakage phenomena may be overlooked.

IV. Conclusion

A novel numerical investigation of mode transitions between bound and leaky states has been presented. It was shown that in a rib waveguide a leaky mode may become bound when a highly lossy substrate, such as a semiconductor, is used. On the other hand, a leaky mode in a strip dielectric guide may evolve into a guided one if the ground plane exhibits ohmic losses as in the case of higher microwave and millimeter-wave frequencies.

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